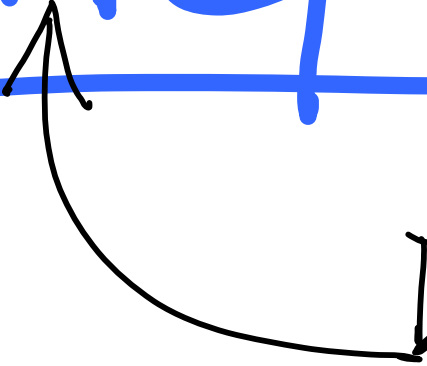


QUADRATURE

PHASE SHIFT

KEYING (QPSK)

Digital
modulation
Scheme.



Consider the following pulses.

$$\underline{P_1(t), P_2(t)}$$

Unit Energy
Signals.

Energy of
both $= 1$
 $E_p = 1$

$$P_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t)$$

$$P_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t)$$

$$0 \leq t \leq T$$

$$T = \frac{K}{f_c}$$

duration $\equiv$ integer # of cycles

$$\sqrt{\frac{2}{T}} \sin(2\pi f_c t + \pi_2)$$

$$= \sqrt{\frac{2}{T}} \cos(2\pi f_c t)$$

Observe $p_2(t)$ is a phase shifted version of $p_1(t)$.

$$\int_{-\infty}^{\infty} p_2^2(t) dt = \frac{2}{T} \int_0^T \sin^2(2\pi f_c t) dt$$

$$= \frac{2}{T} \int_0^T \frac{1 - \cos 4\pi f_c t}{2} dt$$

$$= \frac{2}{T} \cdot \frac{T}{2} - \frac{1}{T} \frac{\sin(4\pi f_c t)}{4\pi f_c} \Big|_0^T$$

$$\boxed{C_p = 1}$$

Inner product,

$$\int_{-\infty}^{\infty} P_1(t) \cdot P_2(t) dt$$

$$= \frac{2}{T} \int_0^T \sin(2\pi f_c t) \cos(2\pi f_c t) dt$$

$$= \frac{1}{T} \int_0^T \sin(4\pi f_c t) dt$$

$$= \frac{1}{T} \cdot \left. \frac{\cos 4\pi f_c t}{4\pi f_c} \right|_0^T$$

$$= \frac{1}{T} \cdot \frac{\cos 4\pi k - 1}{4\pi f_c}$$

$$= 0$$

Inner product

$\Rightarrow$ Pulses are orthogonal.

$$\underbrace{\sqrt{\frac{2}{T}} \cos(2\pi f_c t)}_{P_1(t)}, \underbrace{\sqrt{\frac{2}{T}} \sin(2\pi f_c t)}_{P_2(t)}$$

$\Rightarrow$ Unit Energy + orthogonal.
Form orthonormal basis of signal space.

This is similar to Quadrature
Carrier Multiplexing (QCM).

Analog Modulation
Scheme.

$$m_1(t) \cos(2\pi f_c t)$$

$$-m_2(t) \sin(2\pi f_c t)$$

Carriers are orthogonal.

Two separate message
signals $m_1(t)$, $m_2(t)$
can be modulated on
orthogonal carriers.

$$x(t) = a_1 p_1(t) + a_2 p_2(t)$$

Two symbols can be modulated on $p_1(t)$, $p_2(t)$.

To begin with, each can carry a single bit of information.

Ex: $a_1 = \pm A$ $\leftarrow$ Each carries 1 bit of information
 $a_2 = \pm A$ $\leftarrow$

Average bit Energy = E_b .

$$\Rightarrow \boxed{A = \sqrt{E_b}}$$

$$\Rightarrow x(t) = a_1 p_1(t) + a_2 p_2(t)$$

Carries 2 bits of information

$$x(t) = \begin{cases} A p_1(t) + A p_2(t) \\ A p_1(t) - A p_2(t) \\ -A p_1(t) + A p_2(t) \\ -A p_1(t) - A p_2(t) \end{cases}$$

Set of 4 possible Transmit Signals $x(t)$.

Forms the basis for QPSK or Quadrature Phase Shift Keying.